AP MATHEMATICS
GRADE 12
PRELIMINARY EXAMINATION
11 JULY 2014

MODULE 1
QUESTION 1

For $n=1$: $1^3 + 2(1) = 3$ ✓

and $3 \div 3 = 1$ ✓

$\therefore$ True for $n=1$

Assume true for $n=k$: $k^3 + 2k = 3r$ ✓✓ ; $r \in \mathbb{Z}$

then $k^3 = 3r - 2k$

For $n=k+1$: $(k+1)^3 + 2(k+1)$ ✓

$= (k+1)(k^2+2k+1) + 2k + 2$

$= k^3 + 3k^2 + 3k + 1 + 2k + 2$ ✓

$= (3r - 2k) + 3k^2 + 3k + 1 + 2k + 2$ ✓

$= 3k^2 + 3k + 3 + 3r$ ✓

$= 3(k^2 + k + 1 + r)$ ✓✓ which is divisible by 3.

$\therefore$ True for $n=k+1$, if true for $n=k$ ✓

$\therefore$ True for $n = 1, 2, 3 \ldots$ ✓

[12]

QUESTION 2

2.1 (a) $\log_3 x + \frac{2}{\log_3 x} = 3$ ✓

$\times \log_3 x$: $(\log_3 x)^2 - 3.\log_3 x + 2 = 0$ ✓

$(\log_3 x - 2)(\log_3 x - 1) = 0$ ✓✓

(1)

$\log_3 x = 2$ ✓ or $\log_3 x = 1$ ✓

$x = 3^2$ $\quad$ $x = 3^1$

$x = 9$ ✓ $\quad$ $x = 3$ ✓

(8)

(b) $e^{2x} + 2e^x - 15 = 0$

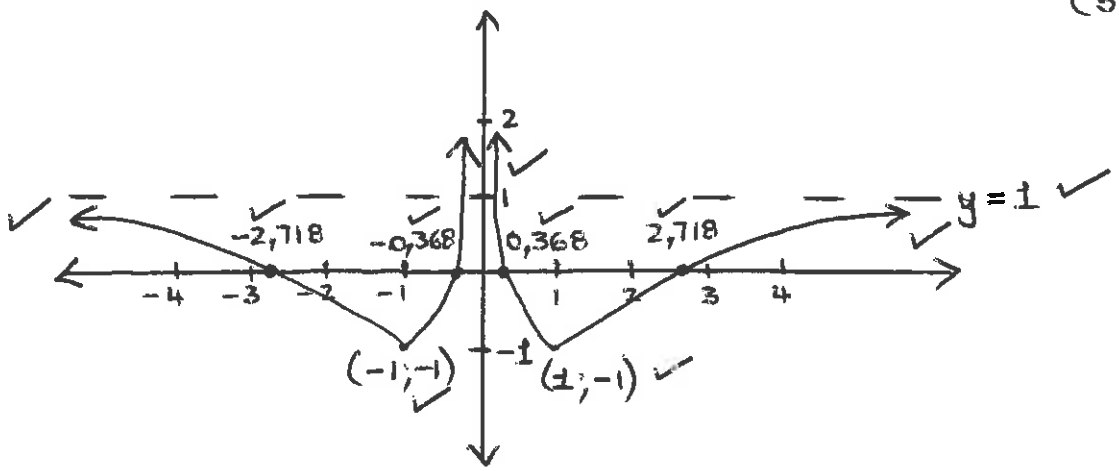
$(e^x + 5)(e^x - 3) = 0$ ✓✓

$e^x \neq -5$ ✓ or $e^x = 3$ ✓

N.A. $\quad$ $x = \ln 3$ ✓

(5)

2.2



(10)

2.3 (a) $g(f(x)) = \dfrac{2}{e^x + 1}$ ✓✓

(2)

(b) $\lim\limits_{x \to \infty} e^x = \infty$

$\therefore \lim\limits_{x \to \infty} \dfrac{2}{e^x + 1} = 0$

As x gets bigger ✓, e^x gets bigger ✓ and therefore $g(f(x))$ get smaller ✓ which makes it a decreasing function.

(3)

(c) $y \in (0; 2)$ ✓✓

(4)

[32]

QUESTION 3

3.1 $$\sum_{n=4}^{7} i^n = i^4 + i^5 + i^6 + i^7 \quad \checkmark\checkmark$$

$$= 1 + i + (-1) + -i \quad \checkmark\checkmark$$

$$= 0 \quad \checkmark\checkmark$$

(6)

3.2 $(x - (3-5i))(x - (3+5i))$ ✓✓

$= [(x-3) + 5i][(x-3) - 5i]$

$= x^2 - 6x + 9 - 25i^2$

$= x^2 - 6x + 34$ ✓✓

$p(x) = 2x^4 + ax^3 + bx^2 - 28x - 34$

$= (x^2 - 6x + 34)(2x^2 - x - 1)$ ✓✓

$-34x$

$+6x$

$-28x$ ✓✓

$= 2x^4 - 13x^3 + 73x^2 - 28x - 34$ ✓✓

$\therefore \quad a = -13$ ✓ and $b = 73$ ✓

OR: $p(x) = 2x^4 + ax^3 + bx^2 - 28x - 34$

$= (x^2 - 6x + 34)(2x^2 + kx - 1)$ ✓✓

$= 2x^4 + kx^3 - x^2 - 12x^3 - 6kx^2 + 6x + 68x^2 + 34kx - 34$

$= 2x^4 + (k-12)x^3 + (67 - 6k)x^2 + (6 + 34k)x - 34$ ✓✓

$6 + 34k = -28$ ✓

$k = -1$ ✓

$\therefore \quad a = -1 - 12$ and $b = 67 - 6(-1)$

$= -13$ ✓ $= 73$ ✓

(12)

[18]

QUESTION 4

4.1
$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) \quad \checkmark\checkmark$$
$$a(2)^2 + 5 = a(2) - 1 \quad \checkmark\checkmark$$
$$4a + 5 = 2a - 1$$
$$2a = -6$$
$$a = -3 \quad \checkmark\checkmark$$

(6)

4.2
$$f'(x) = \begin{cases} -6x \ \checkmark & \text{if } x < 2 \\ -3 \ \checkmark & \text{if } x \geq 2 \end{cases}$$

$$\lim_{x\to 2^-} f'(x) = -6(2) = -12 \quad \checkmark\checkmark$$

$$\lim_{x\to 2^+} f'(x) = -3 \quad \checkmark\checkmark$$

$$\therefore \lim_{x\to 2^-} f'(x) \neq \lim_{x\to 2^+} f'(x)$$
and $\lim_{x\to 2} f'(x)$ d.n.e. $\checkmark$
f is not differentiable at $x = 2$. $\checkmark$

(8)

[14]

QUESTION 5

5.1 (a)
$$LHS = \frac{\sin x}{1-\cos x} - \frac{\cos x}{\sin x} \quad \checkmark$$
$$= \frac{\sin^2 x - \cos x(1-\cos x)}{\sin x(1-\cos x)} \quad \checkmark$$
$$= \frac{\sin^2 x - \cos x + \cos^2 x}{\sin x(1-\cos x)} \quad \checkmark$$
$$= \frac{1 - \cos x \ \checkmark}{\sin x(1-\cos x)}$$
$$= \frac{1}{\sin x} \quad \checkmark$$
$$= \operatorname{cosec} x = RHS \quad \checkmark$$

(6)

(b) $\lim_{x \to 0} (\text{cosec } x)\, x$

$= \lim_{x \to 0} \frac{x}{\sin x}$ ✓

$= 1$ ✓

(2)

5.2 (a) $f(x) = \tan(\cos(3x))$

$f'(x) = \sec^2(\cos(3x)) \cdot (-\sin(3x)) \cdot (3)$ ✓✓✓✓

$= -3 \sec^2(\cos 3x) \cdot \sin 3x$

(4)

(b) $f(x) = \frac{4x}{\sin 4x}$

$f'(x) = \frac{4 \cdot \sin 4x - (\cos 4x)(4)(4x)}{\sin^2 4x}$ ✓✓✓✓✓

$= \frac{4 \sin 4x - 16x \cdot \cos 4x}{\sin^2 4x}$

(7)

5.3 (a) $x^3 - 3xy^2 + y^3 = 1$

$3x^2 - 3 \cdot y^2 + (-3x)(2y)\left(\frac{dy}{dx}\right) + (3y^2)\left(\frac{dy}{dx}\right) = 0$ ✓✓✓✓✓

$\frac{dy}{dx}(3y^2 - 6xy) = 3y^2 - 3x^2$

$\frac{dy}{dx} = \frac{3y^2 - 3x^2}{3y^2 - 6xy}$ ✓

(8)

(b) $m = \frac{3(-1)^2 - 3(2)^2}{3(-1)^2 - 6(2)(-1)} = -\frac{3}{5}$ ✓✓

$y + 1 = -\frac{3}{5}(x - 2)$ ✓✓

$y = -\frac{3}{5}x + \frac{1}{5}$ ✓

(5)

[32]

QUESTION 6

6.1 $\ell = \frac{3}{2}\cos 2x - (x^2 - 2x - 3)$

$= \frac{3}{2}\cos 2x - x^2 + 2x + 3$ ✓

$\ell' = \frac{3}{2}(-\sin 2x)(2) - 2x + 2$ ✓✓✓✓

$-3\sin 2x - 2x + 2 = 0$ ✓

$-3\sin 2x = 2x - 2$

(6)

6.2 Set $f(x) = -3\sin 2x - 2x + 2$ ✓

$f'(x) = -3\cos 2x(2) - 2$ ✓✓

$= -6\cos 2x - 2$

$x_{r+1} = x_r - \frac{-3\sin 2x_r - 2x_r + 2}{-6\cos 2x_r - 2}$

$= x_r - \frac{3\sin 2x_r + 2x_r - 2}{6\cos 2x_r + 2}$ ✓✓

(6)

6.3 $x = 0,25852$ ✓✓✓

(3)

[15]

QUESTION 7

$AB^2 = r^2 + r^2 - 2(r)(r)\cos\theta$ ✓

$= 2r^2 - 2r^2\cos\theta$ ✓

$AB = \sqrt{2r^2(1 - \cos\theta)}$

$= r\sqrt{2 - 2\cos\theta}$ ✓

$S = r\theta$ ✓

$P = r\sqrt{2 - 2\cos\theta} + r\theta$ ✓

$= r\sqrt{2 - 2(1 - 2\sin^2\frac{\theta}{2})}$ ✓ $+ r\theta$

$= r\sqrt{4\sin^2\frac{\theta}{2}}$ ✓ $+ r\theta$

(6) $= r\theta + 2r\sin\frac{\theta}{2}$ ✓

[8]

QUESTION 8

8.1 $f(2,3) \approx 0,96$ ✓✓ (2)

8.2 $f(x) = \frac{(x-4)(x-1)}{(x-3)(x+1)}$ ✓

X-int: $(x-4)(x-1) = 0$

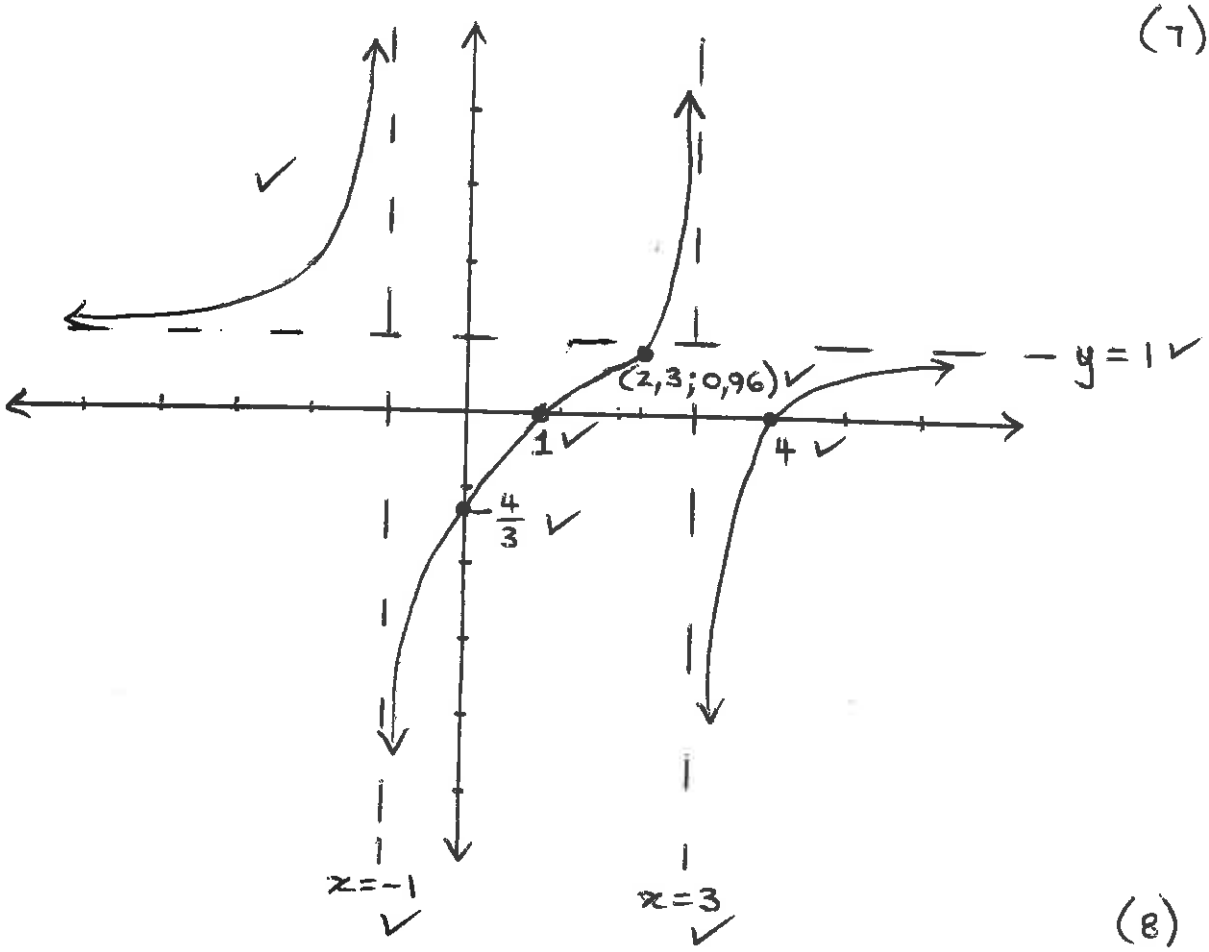
$x = 4$ ✓ or $x = 1$ ✓

Y-int: $f(0) = -\frac{4}{3}$ ✓

Asymptotes: $x = 3$ ✓ and $x = -1$ ✓

$y = 1$ ✓

8.3 (7)



(8)

8.4 $g(x) = \frac{(x-4)(x-1)}{(x-3)(x+1)} \times (x+1)$

$= \frac{x^2-5x+4}{x-3}$ ✓✓

$$\begin{array}{r|l} & x - 2 \quad \checkmark\checkmark \\ \hline x-3 & x^2 - 5x + 4 \\ & -(x^2 - 3x) \\ & \hline & -2x + 4 \\ & -(-2x + 6) \\ & \hline & -2 \end{array}$$ ✓✓

$\therefore g(x) = \frac{-2}{x-3} + x - 2$

$y = x - 2$ ✓✓

(8)

[25]

QUESTION 9

9.1 $\int 2x \cdot \operatorname{cosec}^2(3x^2)\,dx$

$= \frac{1}{3} \checkmark\checkmark \int 6x \cdot \operatorname{cosec}^2(3x^2)\,dx$ ✓✓

$= -\frac{1}{3} \checkmark\checkmark\checkmark \cot(3x^2) + c$ ✓

$= \frac{-\cot(3x^2)}{3} + c$

(8)

9.2 $\int \frac{2x^3 - 3x^2 + 5x}{x}\,dx$

$= \int (2x^2 - 3x + 5)\,dx$ ✓✓

$= \frac{2x^3}{3} - \frac{3x^2}{2} + 5x + c$ ✓✓

(5)

9.3 $\int (3x-1)\sin 3x\,dx$

Set $f(x) = 3x-1$ ✓ and $g'(x) = \sin 3x$ ✓

$f'(x) = 3$ ✓ $\quad g(x) = \frac{-\cos 3x}{3}$ ✓

$$\int (3x-1)\sin 3x\,dx$$

$$= (3x-1)\left(\frac{-\cos 3x}{3}\right) - \int \left(\frac{-\cos 3x}{3}\right)(3)\,dx + c$$

$$= (3x-1)\left(\frac{-\cos 3x}{3}\right) + \int \cos 3x\,dx + c$$

$$= (3x-1)\left(\frac{-\cos 3x}{3}\right) + \frac{\sin 3x}{3} + c$$

(10)

9.4 $\int \sin^2 3x\,dx$

$$= \frac{1}{2}\int (1 - \cos(2(3x)))\,dx$$

$$= \frac{1}{2}\int (1 - \cos 6x)\,dx$$

$$= \frac{1}{2}\left(x - \frac{\sin 6x}{6}\right) + c$$

$$= \frac{x}{2} - \frac{\sin 6x}{12} + c$$

(6)
[29]

QUESTION 10

10.1 $\text{Area} = \int_0^k \frac{2x}{\sqrt{4x^2+3}}\,dx$ ✓

$$= \frac{1}{4}\int_0^k \frac{8x}{\sqrt{4x^2+3}}\,dx$$

$$= \frac{1}{4}\int_0^k 8x(4x^2+3)^{-\frac{1}{2}}\,dx$$

$$\text{Area} = \frac{1}{4}\left[2(4x^2+3)^{1/2}\right]_0^k \quad \checkmark\checkmark$$

$$= \frac{1}{4}\left(2(4k^2+3)^{1/2} - 2(4(0)^2+3)^{1/2}\right) \quad \checkmark\checkmark$$

$$= \frac{1}{4}\left(2\sqrt{4k^2+3} - 2\sqrt{3}\right)$$

$$= \frac{1}{2}\sqrt{4k^2+3} - \frac{1}{2}\sqrt{3} \quad \checkmark\checkmark$$

(9)

10.2 $$\text{Area} = \int_0^{1,5} \frac{2x}{\sqrt{4x^2+3}}\,dx$$

$$= 0,866 \text{ units}^2 \quad \checkmark\checkmark$$

(2)

10.3 $$V = \pi \int_a^b y^2\,dx \quad \checkmark$$

$$= \pi \int_0^{1,5} \left(\frac{2x}{\sqrt{4x^2+3}}\right)^2 dx \quad \checkmark\checkmark$$

$$= \pi \int_0^{1,5} \frac{4x^2}{4x^2+3}\,dx \quad \checkmark$$

(4)

[15]

TOTAL: [200]

MODULE 2

QUESTION 11

11.1 The grade 12s have a higher mean ✓, but they also have a high standard deviation ✓. Random samples have a degree of variability ✓

(3)

11.2 The grade 11s are x and the grade 12s, y.

$H_0 : \mu_x - \mu_y = 0$ ✓

$H_1 : \mu_x - \mu_y < 0$ ✓

Rejection Region: Reject H_0 if $Z < -1,88$ ✓✓

Test Statistic:

$$Z = \frac{7,4 - 8,25}{\sqrt{\frac{(2,15)^2}{45} + \frac{(3,82)^2}{35}}} = -1,1791$$ ✓✓ ✓✓

Conclusion: We can't reject H_0 at the 3% level of significance and have to accept H_0. ✓✓

(10)

[13]

QUESTION 12

12.1 Proportion not in favour = $\frac{158}{550}$ ✓

90% confidence interval $z = 1,645$ ✓✓

$$p = \frac{158}{550} \pm 1,645 \sqrt{\frac{\frac{158}{550}\left(1 - \frac{158}{550}\right)}{550}}$$ ✓✓✓

$[0,2555 ; 0,3190]$ ✓✓ ✓✓

(10)

12.2 (a) $\frac{684,25 + 702,65}{2} = 693,45$ ✓✓

(2)

(b) 98 % CI : $z = \pm 2,33$ ✓

$$2,33 = \frac{693,45 - 684,25}{\frac{\sigma}{\sqrt{550}}}$$ ✓✓✓

$$2,33\left(\frac{\sigma}{\sqrt{550}}\right) = 9,2$$

$$\sigma = 92,60$$ ✓✓

(6)

[18]

QUESTION 13

13.1 $P(B|A) = \frac{P(B \cap A)}{P(A)}$ ✓

$\frac{1}{3} = \frac{P(A \cap B)}{P(A)}$ ✓✓

$\therefore P(A \cap B) = \frac{1}{3} . P(A)$ ✓✓

(5)

13.2 $P(A \cap B) = \frac{3}{8} . P(B)$ ✓✓✓

(3)

13.3 $\frac{1}{3} . P(A) = \frac{3}{8} . P(B)$ ✓

$\therefore P(B) = \frac{8}{9} P(A)$ ✓

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ ✓

$\frac{7}{12} = P(A) + \frac{8}{9} P(A) - \frac{1}{3} . P(A)$ ✓✓✓

$\frac{7}{12} = \frac{14}{9} . P(A)$ ✓

$P(A) = \frac{3}{8}$ ✓✓

(9)

13.4 $P(A \cap B) = \frac{1}{3} \times \frac{3}{8}$ ✓ $= \frac{1}{8}$ ✓✓

(3)

[20]

QUESTION 14

14.1 $$\int_0^2 \left(k - \frac{kx}{3}\right)dx = 1 \quad \checkmark\checkmark$$

$$\left[kx - \frac{kx^2}{6}\right]_0^2 = 1$$

$$\left(2k - \frac{k(2)^2}{6}\right) - 0 = 1 \quad \checkmark$$

$$2k - \frac{2k}{3} = 1$$

$$\frac{4}{3}k = 1$$

$$k = \frac{3}{4} \quad \checkmark\checkmark$$

(7)

14.2 $$\int_0^m \left(\frac{3}{4} - \frac{x}{4}\right)dx = 0,5 \quad \checkmark\checkmark$$

$$\left[\frac{3x}{4} - \frac{x^2}{8}\right]_0^m = 0,5 \quad \checkmark\checkmark$$

$$\left(\frac{3m}{4} - \frac{m^2}{8}\right) - 0 = 0,5 \quad \checkmark\checkmark$$

x-8 : $$m^2 - 6m + 4 = 0 \quad \checkmark$$

$$m \neq 5,2361 \quad \text{or} \quad m = 0,7639 \quad \checkmark\checkmark$$

(9)

[16]

QUESTION 15

15.1 $$P(X = 3) = \binom{12}{3}(0,12)^3(0,88)^9 \quad \checkmark\checkmark\checkmark\checkmark$$

$$= 0,1203 \quad \checkmark\checkmark\checkmark$$

(7)

15.2 $$P(R = 0) = \frac{\binom{25}{0}\binom{15}{6}}{\binom{40}{6}} \quad \checkmark\checkmark\checkmark\checkmark$$

$$= 0,0013 \quad \checkmark\checkmark\checkmark$$

(7)

15.3 Z - value for top 5% is 1,645 ✓✓

$$1{,}645 = \frac{X - 126}{16{,}2} \quad ✓✓$$

$$X = 152{,}649 \quad ✓✓✓$$

(7)
[21]

QUESTION 16

16.1 $$\frac{5! \times (2!)^5}{10!} = 0{,}0011 \quad ✓✓✓✓ ✓✓$$

(6)

16.2 $$\frac{\binom{24}{8}\binom{16}{8}\binom{8}{8}}{3!} \quad ✓✓✓✓$$

$$= 1\ 577\ 585\ 295 \quad ✓✓$$

(6)
[12]

TOTAL: [100]